Electron thermodynamics in the magnetized expansion of a rarefied plasma

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Electron thermodynamics of a plasma thruster plume are crucial to determine propulsive performances and to assess its properties (e.g. electron temperature, far potential drop, and ion direct energy) when impinging far spacecraft surfaces. The work here extends previous analyses to plumes with two electron populations with disparate temperatures. The formation of a quasineutral steepened layer is discussed.

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Introduction

The study of expanding collisionless magnetized plasmas is important to assess the properties of the plasma near the spacecraft surfaces, especially for space plasma thrusters implementing a magnetic nozzle to guide and accelerate the plasma jet, of which are worthmentioning:

1. The helicon plasma thruster (**HPT**)
2. The Electron cyclotron resonance thruster (**ECRT**)
3. The Variable Specific Impulse Magnetoplasma Rocket (**VASIMR**).

View of a ECR Thruster Plume developed by ONERA[8]

The presence of a current-free double layer (**CFDL**) has been reported in some helicon sources[1-3], but the operational conditions that assure this phenomena, as well as the presence of a hot electron tail, are not completely clarified. It is accepted that the CFDL is formed within a plasma that contains two negative species with different temperatures and for a limited range of their density ratio, since Hairapetian and Stenzel[4] demonstrated the direct relation between the presence of hot electrons and the formation of a steepened potential profile.

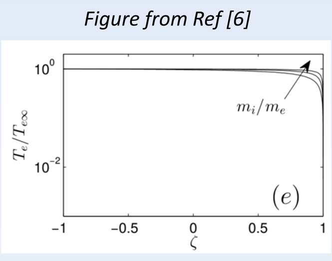
Background: Electron cooling and finite potential drop

In [6], in which this work is based, the authors developed a quasi-1D steady-state model in which particles (mono-energetic ions and Maxwellian electrons) conserved their **magnetic moment** and **energy** along the magnetic field line.

They obtained a finite potential drop and took moments of the electron/ion distribution function to compute plasma properties such as density or electron temperature.

The law obtained for the electron temperature was neither isothermal neither polytropic, but it was concluded that free electrons (or suprathermal electrons) determine the plasma properties far downstream.

Figure from Ref [6]



Background: Two Maxwellian isothermal model [5]

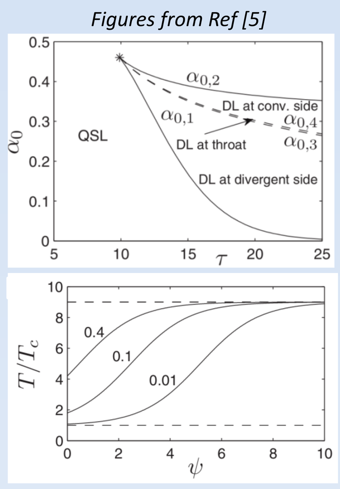
In [5], the author studied a quasi-1D expansion of a fully-ionized collisionless plasma with a hot electron tail. The work was limited to:

1) Two Maxwellian electron populations with two different temperatures and a given density ratio between them
2) Finite nozzles

A parametric analysis of the plasma response was carried out in terms of two parameters of the hot population:

- Upstream relative density $\boldsymbol{\alpha_0} = n_{h0}/n_0$
- Temperature ratio $\boldsymbol{\tau} = T_h/T_c$

Figures from Ref [5]



MAIN CONCLUSIONS

- Anomalous thermodynamic behaviour (**T** increases when **n** decreases).
- All gain in plasma momentum and thrust is related to the supersonic expansion in the divergent nozzle with zero contribution of the double layer.

Objectives of this work

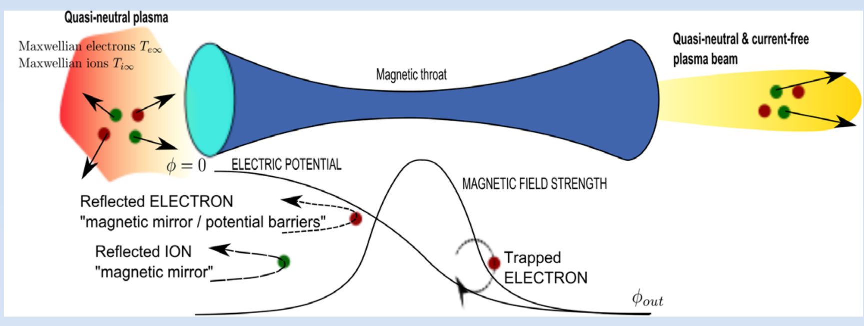
1) Modify the model in [6] so two electron populations (cold and suprathermal) coexist at the source.
2) Study parametrically the plasma response in terms of the upstream density α_0 and temperature ratios Θ_0.
3) Perform a propulsive performance analysis and study the electron cooling phenomena.
4) Compare these results with those obtained in the isothermal model of [5].

Modelling and numerical simulation

Model hypothesis

Model geometry and assumptions:

- Externally applied magnetic field B(z) vanishing at z = ±∞, with a single maximum B_M located at z=0, generating a convergent-divergent magnetic nozzle.
- quasi 1D (paraxial)



- Steady-state
- Collisionless
- All species are fully – magnetized
- Quasineutral
- Current-free
- Particle drift and induced-field effects are totally disregarded

Upstream distribution functions:

- Monoenergetic ions

$$f_i(E_i) = \frac{m_i n_{i0}}{4\pi}\left(\frac{m_i}{2E_0}\right)^{1/2} \delta(E_i - E_0)$$

- Maxwellian cold and suprathermal electrons

$$f_e(E_e) = \alpha \frac{n_{e0}}{(2\pi)^{3/2}}\left(\frac{m_e}{T_{eh0}}\right)^{\frac{3}{2}} exp\left(\frac{-E}{T_{eh0}}\right) + (1-\alpha)\frac{n_{e0}}{(2\pi)^{3/2}}\left(\frac{m_e}{T_{ec0}}\right)^{3/2} exp\left(\frac{-E}{T_{ec0}}\right)$$

Motion invariants:

- Particle energy and Magnetic moment

$$E = \frac{m}{2}\left(w_\parallel^2 + w_\perp^2\right) + q\phi$$

$$\mu = \frac{m w_\perp^2}{2B}$$

Numerical method

The self-consistent solution for the electric potential comes from solving the quasineutrality integral equation,

$$n_i\big(B(z),\phi(z)\big) = n_e\big(B(z),\phi(z)\big)$$

and imposing the zero current condition:

$$\Gamma_i - \Gamma_e = 0$$

Macroscopic variables are computed taking integral moments of the distribution function according to:

$$\langle\chi_\alpha\rangle(z) = \frac{2\pi B(z)}{m_\alpha^2}\iint \chi_\alpha(E_\alpha,\mu_\alpha)\frac{f_{\alpha+}(E_\alpha,\mu_\alpha)}{|w_{\alpha\parallel}|(z)}dE_\alpha d\mu_\alpha$$

$$n_\alpha \equiv \langle 1\rangle, \qquad \Gamma_\alpha = n_\alpha u_\alpha \equiv \langle w_{\alpha\parallel}\rangle, \qquad P_{\alpha\parallel} = n_\alpha T_{\alpha\parallel} \equiv \langle m_\alpha c_{\alpha\parallel}^2\rangle,$$
$$P_{\alpha\perp} = n_\alpha T_{\alpha\perp} \equiv \left\langle\frac{m_\alpha c_{\alpha\perp}^2}{2}\right\rangle = B\langle\mu_\alpha\rangle$$

where boundaries on the integration domain are set according to the type of electron (free or confined, where confined include both forward and backward marching ones).

Inputs:

- Ion to electron mass ratio: $\mathbf{M} = m_i/m_e$
- Ion energy: $\boldsymbol{\varepsilon_i} = E_i / T_{ec0}$
- Upstream electron density ratio: $\boldsymbol{\alpha_0} = n_{eh0}/n_{e0}$
- Upstream electron temperature ratio: $\boldsymbol{\theta_0} = T_{eh0}/T_{ec0}$

M is fixed at 10^4 and $\boldsymbol{\varepsilon_i}$ at 0.1 and a parametric analysis in $\boldsymbol{\alpha_0}$ and $\boldsymbol{\theta_0}$ is carried out.

Constraints:

- The electric potential ϕ(z) is constrained to be a monotonically decreasing function along the magnetic nozzle.
- ϕ(-∞) = 0 (upstream), ϕ(+∞) = ϕ_∞ (downstream)

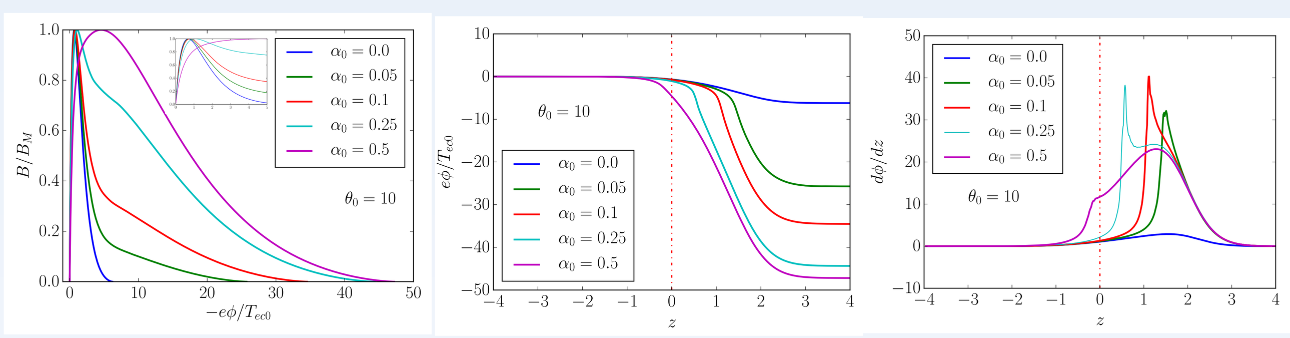
Outputs:

- ϕ(z) , ϕ_∞

An iterative method based on SLSQP techniques is implemented following the scheme presented in [7].

Parametric analysis of the solution

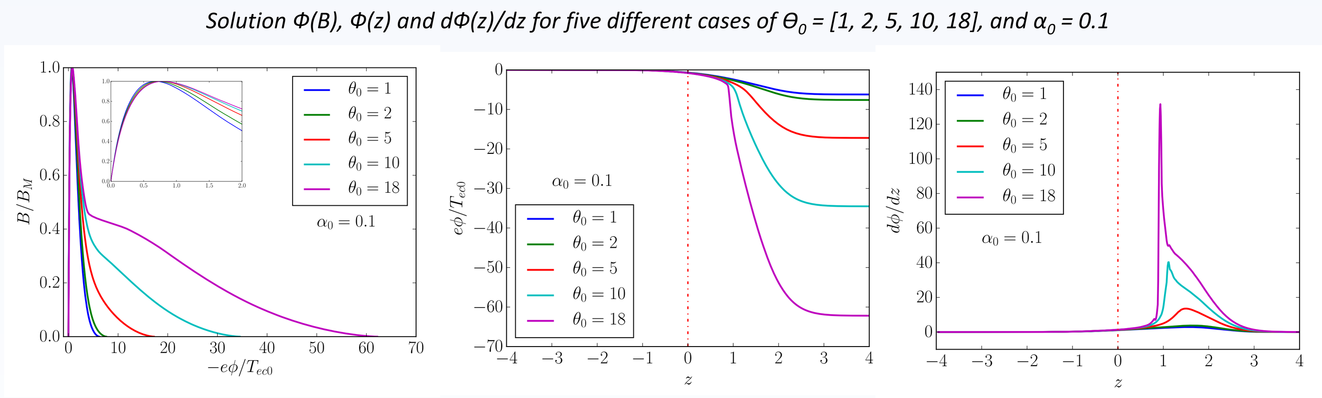
Solution Φ(B), Φ(z) and dΦ(z)/dz for five different cases of α_0 = [0.0, 0.05, 0.1, 0.25, 0.5], and Θ_0 = 10



Parametric analysis in α_0

- Between α_0 = 0 and α_0 = 0.5, a quasineutral steepened layer (QSL) is formed in the electric potential, whose location is closer to the magnetic throat for greater α_0.
- For α_0 > 0.4, the QSL can appear at the convergent side of the expansion. For α_0 > 0.5, the expansion is no more sensitive to α_0 and the discharge is fully dominated by the hot population (without any effect of the cold electrons).
- There is a α_0 threshold for which the effect of suprathermal electrons is more pronounced.
- Although the greatest drop of potential takes place at the divergent region for all cases, the convergent region is also affected by α_0.

Solution Φ(B), Φ(z) and dΦ(z)/dz for five different cases of Θ_0 = [1, 2, 5, 10, 18], and α_0 = 0.1



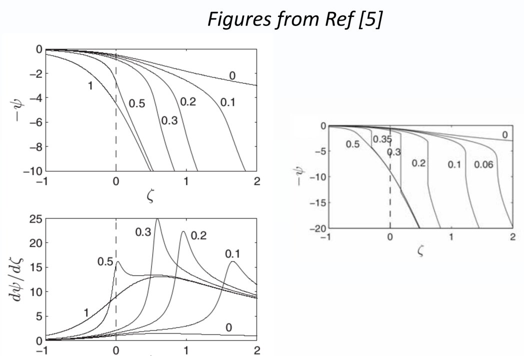
Parametric analysis in Θ_0

- Increasing Θ_0 does not change strongly the location of the QSL, but yes its intensity. This way, what happens before the location of the QSL, is not affected by Θ_0
- There is a Θ_0 threshold (between 2 and 5) in which the effect of the suprathermal electrons becomes important.

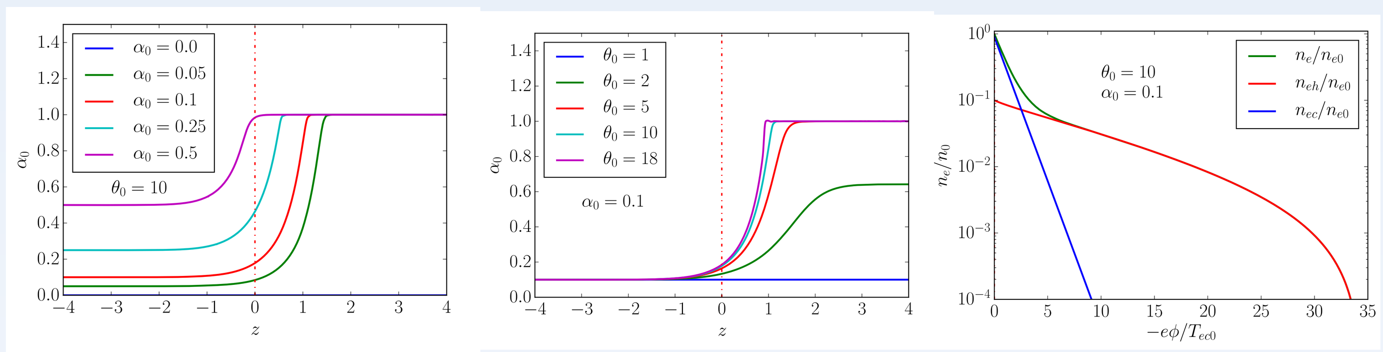
Cross-check with the isothermal fluid model of [5]

The similarities between the results of the two electron isothermal populations in [5] and the results presented here are clearly identified and the leading differences are due to the finite potential drop reached now, against the unbounded potential drop of [5] due to its assumption of Boltzmann electrons.

Figures from Ref [5]



Parametric study of the plasma density



Parametric analysis in α_0

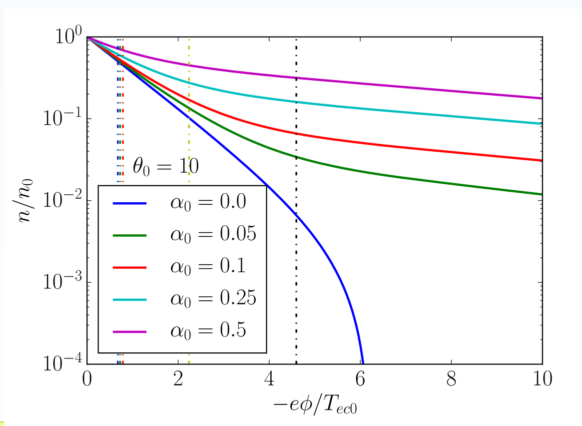
- The α_0 variation changes the location of the QSL (nearer the magnetic throat for greater α_0)

Parametric analysis in Θ_0

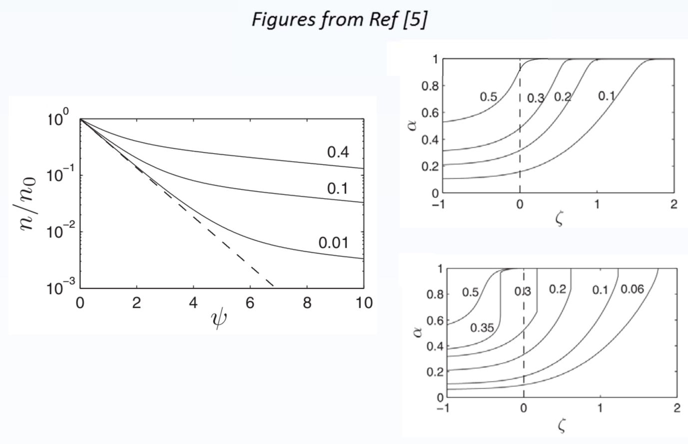
- If Θ_0 is not high enough, the hot population would never dominate the expansion; see the case Θ_0 =2, where about 40% of the final density is given by the cold electrons.
- The slope of α is more pronounced for higher values of Θ_0.
- If the electrons temperature ratio Θ_0 is high enough, the cold electron population is seen as an isothermal ambient plasma (density – electric potential follow the Boltzmann equilibrium relation), since all its contribution to the total density comes from its confined electrons.

Cross-check with [5]

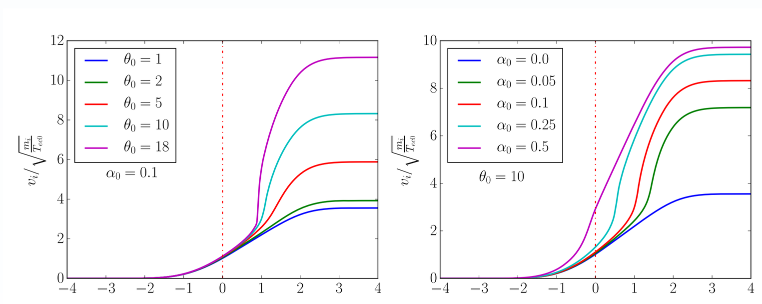
- Unlike [5], the case α_0 = 0 does not follow the Boltzmann relation anymore.



Figures from Ref [5]

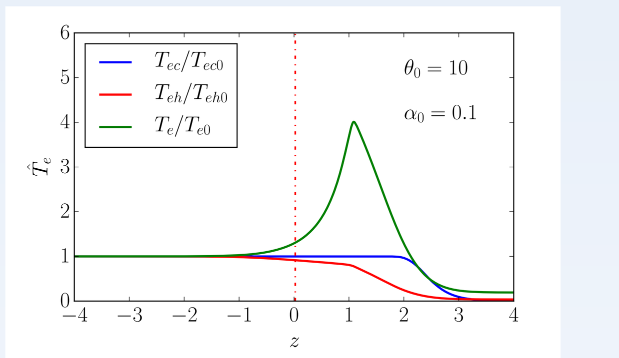


Ion acceleration

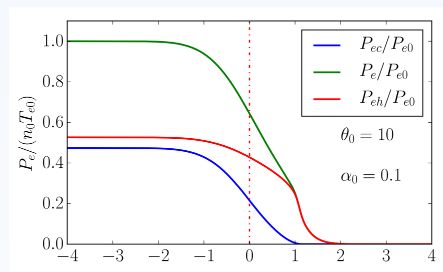


- Increasing the Θ_0 parameter (for small α_0) does not affect the convergent region, but it leads to much higher values of the final ion velocity. The acceleration takes place in a narrower region of the expansion.
- Conversely, the variation of α_0 has already an important effect at the convergent region.

Electron cooling



- The two electron populations suffer an electron cooling effect, although it is more remarkable for the suprathermal ones.
- Although the local temperature for both electron species decreases along the expansion, the global temperature as an average of the local pressure weighted with each species density, experiments a maximum at some point of the divergent region.
- Plasma pressure is clearly dominated by the response of the hot electrons.



CONCLUSIONS

- For 0 < α_0 < 0.5, a quasineutral steepened layer (QSL) is formed. It has a direct impact on the final electric potential drop, and therefore in the final ion velocity.
- By increasing in Θ_0 the effect of the QSL becomes stronger, but it does not change its location. Numerically, it has not been possible to simulate cases with Θ_0 > 20, since it is difficult to preserve the quasineutrality condition.
- Macroscopic plasma variables, such as density or temperature, are dominated by the cold electrons, and then the control of the hot electrons arises.
- In relation with the two isothermal electron species studied in [5], results are qualitatively consistent. The most significant differences encountered are due to the bounded electric potential drop.

References

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