Analysis of Thermionic Bare Tether Operation Regimes in Passive Mode

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The Thermionic Bare Tether (TBT)[1]

<u>TBT Concept and operation</u>

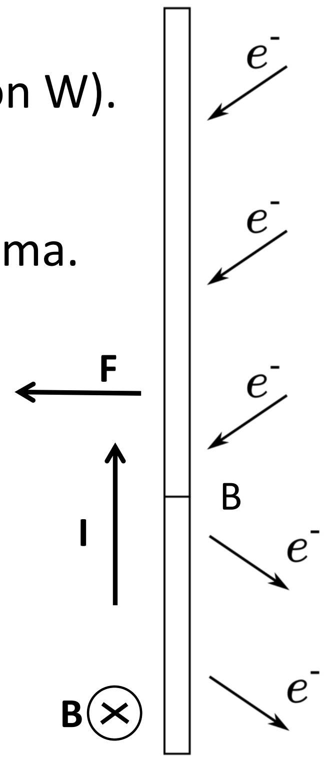
- A TBT is a tape of aluminum coated with a thermionic material (low work function W).
- Typical length/width/thickness = few km/few cm/tens microns.
- Each TBT segment <u>naturally</u> captures (emits) electrons from the ionospheric plasma.
- Geomagnetic field (**B**) exerts a Lorentz drag (**F**) on the tether current (**I**).

<u>Main Application: space debris deorbiting</u>

- No power supply, no propellant (fully passive device).
- As simple as a drag sail but (about) 100 times more efficient.

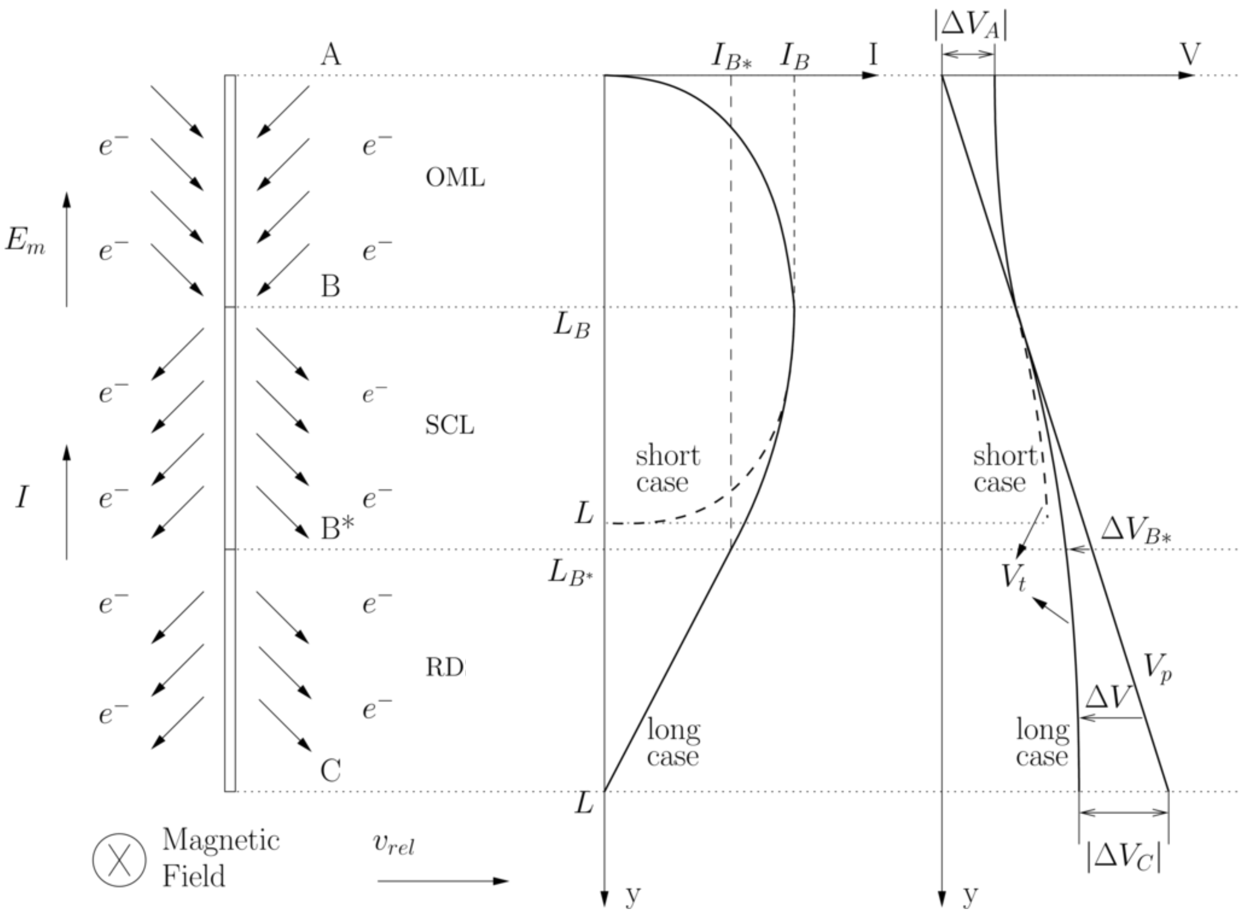
<u>However,</u>

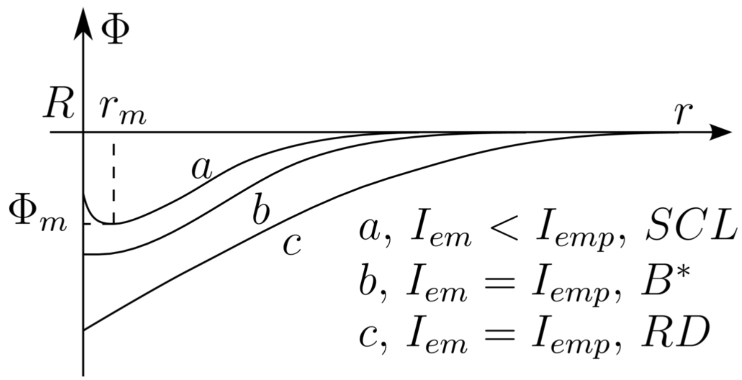
- TBT/plasma modelling - > How much current does circulate along a BTT ?.
- Material issues are opened -> Coating[2] of [Ca24Al28O64]4+(e-)4



1] Williams, John D., Juan R. Sanmartin, and Lauren P. Rand. *Plasma Science, IEEE,* (2012): 1441-1445.
2]Toda, Yoshitake, et al. *Applied Physics Letters* 87.25 (2005), 4103.

TBT Operation and Space Charge Effect





TBT Models

TBT Current/Voltage Model

$$\frac{d\Delta V(s)}{ds} = \frac{I(s)}{\sigma_c A_t} - E_m$$

$$\frac{dI(s)}{ds} = \begin{cases} p_t \times j_{OML} & 0 < s < s_B \\ -p_t \times j_{SCL} & s_B < s < s_{B*} \\ -p_t \times j_{RD} & s_{B*} < s < s_c \end{cases}$$

with I(0) = I(L) = 0, $\Delta V(s_B) = 0.$

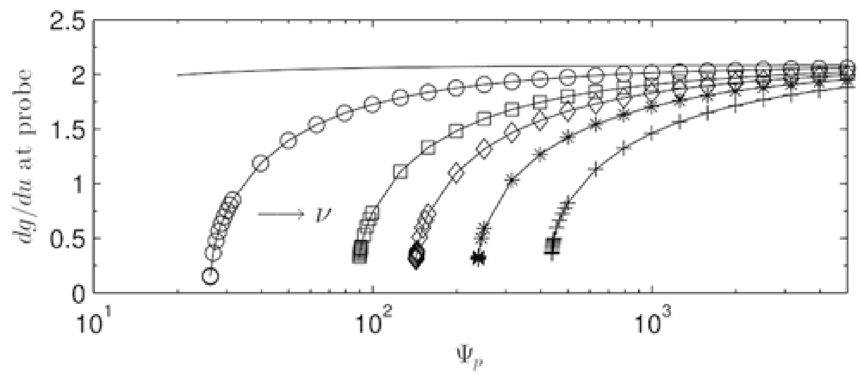
Collection/Emission laws

$$j_{OML} = \frac{eN_\infty}{\pi}\sqrt{\frac{2e\Delta V}{m_e}}$$

$$j_{SCL} = f \times j_{RD} \qquad f < 1$$

$$j_{RD} = \nu e N_\infty \sqrt{\frac{2kT_P}{\pi m_e}} \qquad \nu = N_{emp}/N_\infty$$

Asymptotic analysis of Emissive probe[1]



$$\frac{e|\Delta V_{B*}|}{kT} \approx 0.38\,\nu^{1.4}$$

$\nu = 20, 50, 70, 100, 150,$ $\quad T_e = T_i = T = 4T_p$

There are two limit regimes

- Negligible SCL emission regime.
- Short Tether Regime (no RD segment)

1]Chen, Xin, and J. R. Sanmartín. Physics of Plasmas (2015), 053504.

Negligible *space-charge-limited* emission regime

1) Integrating the equation for bias from B to B^*, with $\Delta V_B = 0$ and $I(s)$ maximum at B, yields

$$\frac{d\Delta V}{ds} = \frac{I}{\sigma_c A_t} - E_m = -E_m \left[1 - \frac{I(s)}{\sigma_c E_m A_t}\right], \qquad (0 < s < \mathrm{L}) \tag{1}$$

$$\Rightarrow \quad -\Delta V_{B*} = E_m \int_{s_B}^{s_{B*}} ds \left[1 - \frac{I(s)}{\sigma_c E_m A_t}\right] > E_m \left[1 - \frac{I_B}{\sigma_c E_m A_t}\right](s_{B*} - s_B)$$

$$\Rightarrow \quad \left(1 - i_B\right)(\tilde{s}_{B*} - \tilde{s}_B) < \left|\Delta V_{B*}\right| / E_m L \qquad (i \equiv I / \sigma_c E_m A_t, \quad \tilde{s} \equiv s / L) \tag{2}$$

The above bias ratio at tether point B^* is very small under a broad range of conditions. Analysis of just full RD-emission can give a corresponding ratio $e|\Delta V|/kT$ at SCL current onset. For $\nu = 20, 50, 70, 100$ (and $T_i = T_e \equiv T = 4\, T_p = 0.1\, eV$), it gave values 26.2, 89.7, 143.0, 238.8 [2], or roughly,

$$e\left|\Delta V_{B*}\right| / kT \approx 0.38\, \nu^{1.4} << eE_m L / kT \tag{3}$$

as applying to values $E_m L = 450$ V, $kT = 0.1\, eV$, say, i.e. $eE_m L / kT = 4500$. Small B^* bias-ratio thus requires not too low E_m or too high $\nu \equiv N_{emp} / N_\infty \propto T_p^{3/2} \exp(-W / kT_p) / N_\infty$.

For $1 - i_B$ of order unity (*moderate ohmic effects*, see later), Eq.(2) proves the BB^* segment short.

2) Consider now current exchange in the segment from B to C. Full *Richardson-Dushman* emission applies from B^* to C. Using $I_C = 0$ then yields

$$\frac{dI}{ds} = - p_t \times j_{RD} = - p_t \times v e N_\infty \sqrt{\frac{2kT_p}{\pi m_e}}, \quad (s_{B*} < s < L) \qquad \Rightarrow I_{B*} = p_t v e N_\infty \sqrt{\frac{2kT_p}{\pi m_e}} (L - s_{B*}) \tag{4}$$

In the segment from B to B^*, emission is reduced by *space-charge*, leading to

$$\left|\frac{dI}{ds}\right| = - \frac{dI}{ds} < p_t \times v e N_\infty \sqrt{\frac{2kT_p}{\pi m_e}}, \qquad (s_B < s < s_{B*})$$

$$\Rightarrow \quad -\left(I_{B^*} - I_B\right) = \int_{s_B}^{s_{B*}} ds \left|\frac{dI}{ds}\right| < p_t \times v e N_\infty \sqrt{\frac{2kT_p}{\pi m_e}} (s_{B^*} - s_B), \tag{5}$$

Use of (4) and (5) yields

$$\frac{i_B - i_{B*}}{i_{B*}} < \frac{\tilde{s}_{B*} - \tilde{s}_B}{1 - \tilde{s}_{B*}}, \tag{6}$$

showing that current varies little in the mid-segment and thus requires no *space-charge-limited* emission modelling [3], unless segment B^*C is similarly short (see later).

3) Standard *bare-tether* analysis involves Eq.(1) for $d\Delta V/ds$ and the anodic *OML* collection rate

$$\frac{dI}{ds} = \frac{p_t}{\pi} e N_\infty \sqrt{\frac{2e\Delta V}{m_e}}, \qquad (0 < s < s_B). \tag{7}$$

Both equations are then rewritten in terms of dimensionless variables, i and

$$\xi \equiv s/L^*, \quad \psi \equiv \Delta V/E_m L^*, \tag{8a, b}$$

where L^*, gauging ohmic and collection impedances, satisfies the identity

$$\left(\frac{L}{L^*}\right)^{3/2} \equiv \frac{4}{3\pi} \frac{p_t eN_\infty L}{\sigma_c E_m A_t} \sqrt{\frac{2\,eE_m L}{m_e}}. \tag{9}$$

One then readily finds exact results, particularly convenient here, giving $i_B\left[\psi_A(L\tilde{s}_B/L^*)\right]$,

$$1-\psi_A^{3/2} = (1-i_B)^2, \qquad \xi_B \equiv \frac{L}{L^*}\tilde{s}_B = \psi_A \times \int_0^1 \frac{d\varphi}{\sqrt{1-(1-\varphi^{3/2})\psi_A^{3/2}}}. \tag{10a, b}$$

Using Eq.(9) in (4), and $\tilde{s}_{B*} \approx \tilde{s}_B$, and $i_{B*} \approx i_B$, yields a relation determining $\tilde{s}_B\left(L/L^*,\ kT_p v^2/eE_m L\right)$,

$$i_{B*} = \frac{3\sqrt{\pi}}{4}\left(\frac{L}{L^*}\right)^{3/2} \times v\sqrt{\frac{kT_p}{eE_m L}} \times (1-\tilde{s}_B) \approx i_B\left[\psi_A(L\tilde{s}_B/L^*)\right] \tag{11}$$

Moderate ohmic effects require, say, $i_B \approx i_{B*} < 0.5$ in (11):

1) The ratio $(L/L^*)^{3/2} \propto p_t N_\infty / A_t E_m$ in Eq. (9) shows that such condition might best apply to a round tether as against a tape one, due to its low *perimeter / cross-section* ratio, p_t / A_t, particularly if operating in Jupiter, because of its low *plasma density/ motional- field* ratio, N_∞ / E_m.

2) In general, the low factor $\nu\,(kT_p / eE_m L)^{1/2}$ values here considered for negligible *SCL* emission also help keeping ohmic effects moderate.

Particularly simple is the weak ohmic-effects case. Just setting i_B and ψ_A small in Eqs. (10a, b) gives the approximations

$$2 i_B \approx \psi_A^{3/2} \approx (L / L^*)^{3/2} \tilde{s}_B^{3/2} \qquad (12a, b)$$

finally leading in (11) to a simple equation for s_B / L, explicitly showing that low $\nu\,(kT_p / eE_m L)^{1/2}$ values also help keeping segment B^*C long enough,

$$\frac{1 - \tilde{s}_B}{\tilde{s}_B^{3/2}} \approx \frac{2\sqrt{eE_m L / kT_p}}{3\sqrt{\pi}\,\nu} \qquad (13)$$

$\Rightarrow$ For the *RHS* of Eq. (13) large, $1 - \tilde{s}_B$ must be of order unity.

***Short tether* (no full-*RD* emission) regime**

Again integrating for bias in Eq. (1) allows writing

$$-\Delta V_{B*} = E_m \int_{s_B}^{s_{B*}} ds \left[1 - \frac{I(s)}{\sigma_c E_m A_t}\right] = E_m \left[1 - <\frac{I}{\sigma_c E_m A_t}>_{BB*}\right](s_{B*} - s_B)$$

$$\Rightarrow \quad \left|\Delta V_{B*}\right| / E_m L = \left[1 - \langle i \rangle_{BB*}\right] \times (\tilde{s}_{B*} - \tilde{s}_B), \qquad (14)$$

where $< >_{BB*}$ stands for averaging over the segment *BB**.

As the ratio in the *LHS* of (14) increases with ν, at given $eE_m L / kT$, the *B*C* segment collapses and i_{B*} decreases to zero along with $L - s_{B*}$. Assuming (3) still valid, Eq. (14) then takes the form

$$\frac{0.38\,\nu^{1.4}}{eE_m L / kT} \approx \left[1 - \langle i \rangle_{BB*}\right] \times (1 - \tilde{s}_B). \qquad (15)$$

We note that the *RHS* of Eq. (5), where we now set $i_{B*} = 0$ and $s_{B*} = L$, overestimated emission by having used the full *Richardson-Dushman* current density law. Introducing some appropriate factor $f < 1$ in (5), which allows equating it to i_B as given by (10a, b), and using Eq. (9) in (5), yields

$$i_B\left[\psi_A (L\tilde{s}_B / L*)\right] = f \times \frac{3\sqrt{\pi}}{4}\left(\frac{L}{L*}\right)^{3/2} \times \nu \sqrt{\frac{kT_p}{eE_m L}} \times (1 - \tilde{s}_B). \qquad (16)$$

In the simple case of weak ohmic effects, Eqs. (15) and (16) simplify, respectively, to

$$\frac{0.38\,\nu^{1.4}}{eE_m L/kT} \approx 1 - \tilde{s}_B\,, \qquad \tilde{s}_B^{\,3/2} \approx f \times \frac{3\sqrt{\pi}}{4}\,\nu\,\sqrt{\frac{kT}{eE_m L}} \times \left(1 - \tilde{s}_B\right). \qquad (17a, b)$$

Using Eqs. (17a, b) to determine $\tilde{s}_B^{\,3/2}$, and then using it in (17a), finally yields a *short tether* condition,

$$\frac{eE_m L}{kT} \approx 0.38\,\nu^{1.4} + 0.63 \times f^{2/3}\,\nu^{1.6}. \qquad (18)$$

For given ν, there is no *RD* emission segment for eE_mL/kT below the value in the *RHS* of Eq. (18).

To determine f, we model *SCL* emission in the cathodic segment as a double layer involving emitted electrons and attracted ambient ions (neglecting ion current and ambient electrons). This is simplified by using the *sheath* radius in OML ion collection as the *anodic* radius in the classical analysis [4] for emission in vacuum from an inner cylinder to a coaxial anodic one [5], [6].

In absence of ohmic effects, the above leads to a current density linear in distance to point B,

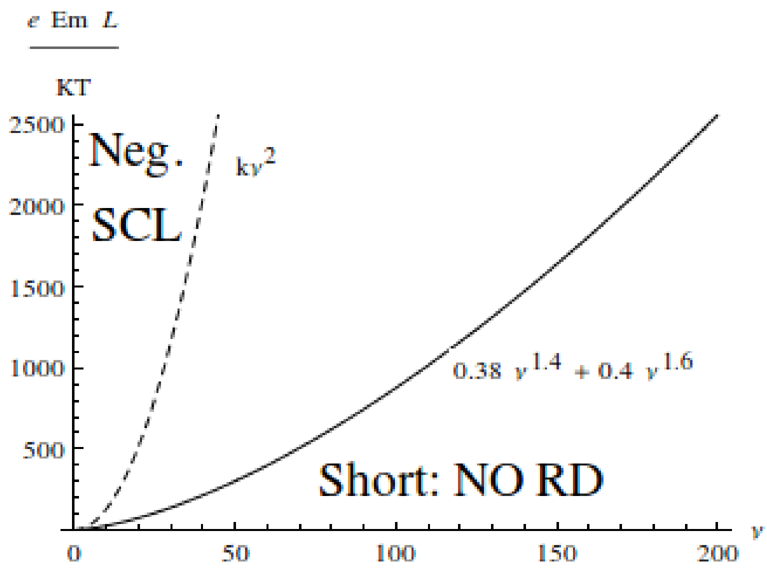
$$\frac{dI}{ds} = -\,p_t \times j_{RD} \times \frac{s - s_B}{L - s_B}$$

for the *short tether* condition, leading to $f = 0.5$ in Eq.(18), which becomes

$$\frac{eE_m L}{kT} \approx 0.38\,\nu^{1.4} + 0.40\,\nu^{1.6} \qquad \Rightarrow \qquad \nu \approx 288 \quad \text{for} \quad eE_mL/kT = 4500.$$

Conclusions

- TBT's underlying physics is robust and simple but its modelling is a hard task.
- The analysis did not require an explicit law for $j_{SCL,}$ but required the potential bias at SCL to RD transition .
- Parametric conditions to operate a TBT in two limit regimes were derived.
- Future works:
  - Broad parametric analyses for several probe radius and probe-to-electron temperature ratios.
  - Studies combining longitudinal tether equations with full 2-dimensional solutions of the Vlasov-Poisson system.



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