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A Statistical Analysis of the Worst GEO Plasma Environment ー Extreme Value Analysis of the High Electron Temperatures ー

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Introduction

- Statistical analyses of the worst plasma environment on the geosynchronous Earth orbits (GEO) are important for geostationary spacecraft designs, because it frequently causes satellite anomalies due to electrostatic discharges induced by surface charging.
- A key parameter of the surface charging is high electron temperature.
- We analyze upper limits and occurrence frequencies of the parallel and perpendicular high electron temperatures using an extreme value analysis.

LANL MPA Key Parameter Data

- Magnetospheric Plasma Analyzer (MPA) Key Parameter data observed by Los Alamos National Laboratory (LANL) geostationary satellites (L9, L0, L1, L4, L7, A1, A2) in 1992-2008 are published on the NASA Goddard Coordinated Data Analysis Web (CDAWeb).
- The MPA Key Parameter data provide parallel and perpendicular temperatures and densities for electrons and ions and spacecraft potential at approximately every 86 seconds.
- The total number of their data points is 14,230,365 which are approximately equivalent to the 35-year data.

Original L1 data : $T_e$para and $T_e$perp (0.03-45 keV/q)

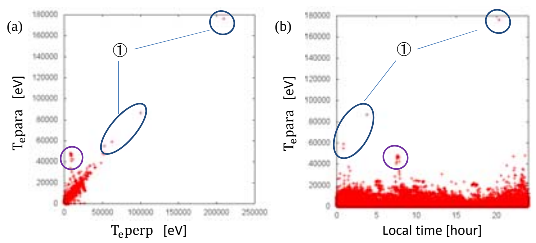

Fig. 1. Scatter plots of the relationship (a) between the perpendicular and parallel electron temperatures and (b) between the local time and the parallel electron temperature for the L1 data.

TABLE I. Parameters of invalid data points

	dd.mm.yyyy	hh.mm.ss	$T_e$para [eV]	$T_e$perp [eV]	$T_i$para [eV]	$T_i$perp [eV]	S/C Potential [Volts]
①	17-03-2003	07:23:08	176207.0	209557.0	0	0	-285900
	29-05-2003	14:52:41	86731.7	100189.0	0	0	-133000
	27-02-1997	20:35:10	59130.5	62933.7	0	0	-81790
	27-02-1997	20:33:44	55096.1	52717.2	0	0	-69990
○	11-09-1995	03:10:41	48351.7	8441.06	5831.95	6763.98	-32940
	11-09-1995	03:13:33	47805.4	8812.98	5784.50	6753.73	-32940
	11-09-1995	03:09:15	47804.3	8519.50	5810.11	6854.55	-32940
	11-09-1995	03:14:59	47592.7	8938.47	5748.21	6825.42	-32940

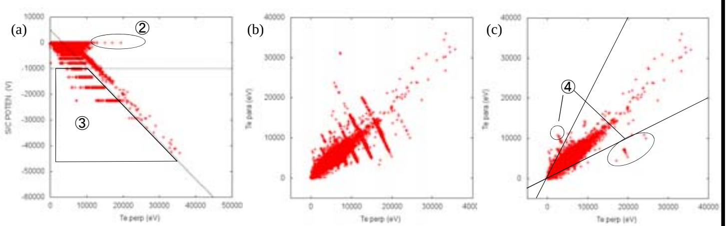

Fig. 2. Scatter plots of the relationship (a) between the perpendicular electron temperature and spacecraft potential and (b) between the perpendicular and parallel electron temperatures after eliminating data points of ① for the L9 data. Plot (c) is the same plot as (b) after eliminating data points of ①, ②, and ③.

- Invalid data criteria

① Ion density or ion temperatures are null. (Fig. 1)

② Spacecraft potential is null while $T_e$para or $T_e$perp is lager than 10,000 eV. (Fig. 2(a))

③ Spacecraft potential is less than -10,000 V and $T_e$perp is under a line ($T_e$perp < (5,000-spacecraft potential)/1.45 for the L9 data). (Fig. 2(a))

④ $T_e$para > 2 $T_e$perp or 2$T_e$para < $T_e$perp of the groups of the data points distributed at some particular spacecraft potential values (as seen line segments in fig. 2(a)(b)) when spacecraft potential is less than -1 V. (Fig. 2(c))

- We eliminate 9,089 data points as invalid data points, which are 0.064% of the total data points, and make a corrected data set. (Fig. 3(a))

Extreme Value Analysis

Peak-Over-Threshold model

- Assuming that the data set $X = \{x_i\}$ is the independently and identically distributed random variables that are governed by the distribution function F(x).
- Conditional (cumulative) distribution function $F(X|X>\mu)$ is for the data subset $\{x_1, x_2, \ldots, x_n\}$ over a fixed threshold μ.
- Assuming that $F(X|X>\mu)$ approaches the generalized Pareto distribution (GPD) if μ becomes sufficiently large.

Generalized Pareto distribution (GPD)

$$F(X|X>\mu) \sim W_{\mu,\sigma,\gamma}(x) = 1-\left(1+\gamma\frac{x-\mu}{\sigma}\right)^{-\frac{1}{\gamma}}$$

μ : location parameter, γ : shape parameter, σ : scale parameter

- $\gamma < 0$: Pareto distribution II (finite upper limit, $x < \mu - \sigma/\gamma$)
- $\gamma > 0$: Pareto distribution (heavy tail, $x < \infty$)
- $\gamma = 0$: Exponential distribution (thin tail, $0 < x < \infty$)

- First, we calculate μ by a common method used to examine a mean excess function $M(u) = E[X-u \mid X>\mu]$, which represents the mean value of the exceedance data subset over a threshold u.
- The GPD characteristic indicates that $M(u) = (\sigma+\gamma u)/(1-\gamma)$ is a linear function of u. The parameter u can be identified by the minimum u that conforms to this linearity.
- $M(u)$ of the corrected data set appears to be linear beyond u> 22000 as seen in fig. 3(b). Thus the data subset of $T_e$para > μ= 22,000 eV can be expected to obey an identical distribution to that represented by the form of the GPD.

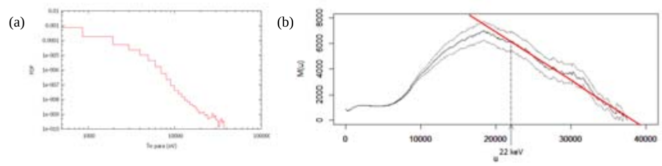

Fig. 3. (a) probability density function (PDF) of the corrected data set and (b) the mean excess function of electron temperature as a function of the threshold u.

- The other parameters (γ, σ) are estimated by conventional maximum likelihood methods for μ=22,000 and shown in table II. (We use the R package "ismev.")

TABLE II. Parameters for parallel and perpendicular electron temperature

	μ [eV]	γ	σ [eV]
$T_e$para	22000	-0.668 ± 0.0734	10527 ± 1037
$T_e$perp	22000	-0.482 ± 0.0561	9357 ± 813

- The results show that the electron temperatures have an upper limit and the occurrence frequency per year for $T_e > x$ can be written as

$$y = \frac{mk}{n}\left(1+\gamma\frac{x-\mu}{\sigma}\right)^{-1/\gamma}$$

k : number of the data > x
m : number of the data per year
n : total number of data

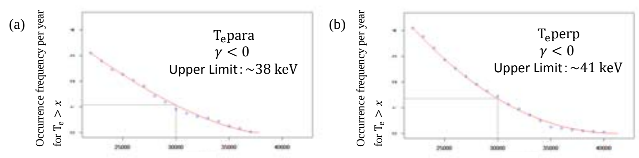

Fig. 4. Plot of occurrence frequency per year (a) for $T_e$para $> x$ and (b) for $T_e$perp $> x$.

Summary

- It requires careful attention to use the LANL data on the CDAWeb for statistical analyses of the worst GEO plasma environment.
- We can estimate upper limits and occurrence frequencies of the parallel and perpendicular high electron temperatures using the extreme value analysis.
- The results are useful for the GEO spacecraft designs to estimate surface charging occurrence frequency.

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