Semi-analytic and PIC (Particle-in-cell) Methods for Quantifying Charging in Dense, Cold Plasma

V.A. Davis, M.J. Mandell, D.L. Cooke, D.C. Ferguson

Abstract—Researchers use several techniques for the calculation of sheath structure and surface currents in short Debye length plasma for the purpose of calculating surface potentials of complex spacecraft. These techniques include the assignment of analytic results for simple geometries, semi-analytic methods that use Poisson's solution with an analytic charge density formula in conjunction with particle tracking, hybrid-PIC (particle in cell) techniques in which the charge density is determined by tracking ions and assuming Boltzmann electron densities, and full PIC. Hybrid-PIC and full PIC techniques can be applied to both static and dynamic plasmas.

We compare the various techniques available in the plasma modeling code *Nascap-2k*, along with the analytic approach used in the analysis tool EWB. We review the strengths, weaknesses, and limitations of the available models. In the appropriate limits, each approach gives the analytic result—within the accuracy of the calculation. When used under conditions outside the limits of the approximations, results are not reliable and may be misleading. At the same time, the PIC and hybrid PIC approaches can be misleading when phase space is not adequately sampled.

When used within their range of applicability, these techniques are powerful tools in assessing charging in short Debye length plasma.

Keywords—charging; modeling; dense plasma

I. Charging Mechanisms

This paper surveys the various approaches used to model spacecraft charging in cold, dense plasma, such as found in low-Earth-orbit. The range of plasma properties under consideration are listed in Table I. In these plasmas, with a few exceptions (such as when thin dielectrics lead to high surface capacitances), the current is high enough that spacecraft surface potentials adjust to changes in the spacecraft and environment within milliseconds. Dynamic effects are associated with wake effects, result from switching of solar arrays on and off, and occur in dynamic experiments. However, the main interest, and the focus of this paper, is the calculation of steady-state potentials.

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This research was carried out under contract with the Air Force Research Laboratory, FA9453-11-C-0262

TABLE I. Plasma properties

Temperature	0.1 to 5 eV
Density	10^9 to 10^{12} m^{-3}
Debye length	0.2 to 50 cm
Electron thermal current	9 µA m^{-2} to 60 mA m^{-2}
Oxygen thermal current	0.05 µA m^{-2} to 0.3 mA m^{-2}
Hydrogen thermal current	0.2 µA m^{-2} to 1 mA m^{-2}
Ram current	1 µA m^{-2} to 1 mA m^{-2}

In low energy plasma, the net surface current is dominated by the incident charged particles. This is different from higher energy, more tenuous plasmas, such as geosynchronous substorm and auroral environments. In cold plasma the energy of the incident charged particles is low, so few secondary electrons are created, and (except in the more tenuous environments) even the ion thermal current exceeds the photoemitted electron current.

In low-Earth orbit the spacecraft velocity is on the order of 7 km/sec, slow compared with the electron thermal speed, but fast compared with the thermal speed of an atomic oxygen ion. The ion and electron densities immediately behind the spacecraft are dramatically reduced because only ions with velocities comparable to the spacecraft velocity can exist in this "deep wake" region. The electron density would be near uniform were it not for the negative potential due to electron space charge from the excess electrons. In this mesothermal plasma, the ion current, and to a much lesser extent the electron current, depends on the angle of the surface with respect to the motion.

Significant charging in cold plasma is driven by spacecraft-generated voltages. The resulting potentials are of the same order. A common example of spacecraft-generated voltages is the positively biased interconnects and bus bars of the solar arrays that are exposed on most spacecraft. These electrodes collect electrons from the surrounding plasma, driving the spacecraft ground negative, generally to about 90% of the entire solar-array-generated voltage. The floating potential is determined by the ion current to other exposed conductive surfaces. The electron current collected by all the positive potential conducting surfaces (such as the interconnects) is balanced by the ion current collected by the negative potential conducting surfaces.

In the absence of spacecraft-generated potentials and on insulating surfaces, the surface potential is determined by the balance between the attracted species and the repelled species. Thus surfaces float negative a few times the plasma temperature, so that the attracted ion current is balanced by the repelled electron current. Under unusual conditions, potentials on the order of the ion kinetic energy can develop in order to repel the ion flux.

II. Available Techniques

The closely related techniques of current balance and charging are used to compute the equilibrium potential of surfaces in cold, dense plasma. In current balance, the net current as a function of surface potentials is computed in order to find the zero of the current-voltage relation. When the charging technique is used, initial surface potentials are assumed. The resulting surface currents and an assumed capacitance are used to determine the change in surface potentials. The process is then iterated until the zero current condition is reached. In both approaches, the current as a function of surface potential is the central parameter.

The current computations depend on how the plasma is modeled. There are four basic approaches, along with several variations.

- Analytic currents
  - Currents computed using a formula
- Particle Tracking/Analytic space charge
  - Poisson's equation solved with space charge specified by an analytic formula
  - Current of one or both species computed by tracking macroparticles.
- Hybrid PIC
  - Poisson's equation solved with space charge given by ion densities from macroparticle tracking and electron densities from an analytic formula.
  - Current of one or both species computed by tracking macroparticles
- PIC

Each of these approaches has benefits and drawbacks. While analytic approaches are faster and more stable, the underlying approximations may or may not be applicable. Self-consistent PIC calculations include all the physical phenomena, but require care to adequately represent the phase space distribution function and are computationally intensive.

A. *Analytic currents*

In this approach, formulas for plasma currents are derived by assuming an idealized geometry and plasma and calculating an exact answer. A major advantage is that once a formula has been developed, applying it to a new system is quick and easy. The applicability of a formula to other conditions is limited by how close the system under consideration is to the ideal. Generally these formulas depend on local values such as the surface potential and orientation with respect to the direction of motion, and sometimes the surface electric field. Here we discuss two common and one not-so-common approximations. In the plasmas of interest, in the absence of spacecraft driven changing surface potentials, it is reasonable to assume that the plasma is at equilibrium and therefore, is Maxwellian.

1) *Planar*

In the absence of motion, if a negative potential collecting surface is very large compared to the Debye length, the attracted current is the ion thermal current and the repelled current is the electron thermal current reduced by the Boltzmann factor. This condition can be used to determine the floating potential of an isolated object.

$$ne\sqrt{\frac{eT}{2\pi m_i}} = ne\sqrt{\frac{eT}{2\pi m_e}}\,exp\left(\frac{\phi}{T}\right) \qquad (1)$$

For an oxygen plasma, the solution to this equation is $\phi = -5.14\ T$, five times the temperature.

When the collecting surface is in motion, the thermal current is multiplied by a velocity dependent angular factor on the ram side. The simplest approximation is to use the ion ram current times a cosine of the angle with respect to flow for the ion current and the same factor as above for the electron current.

$$nev_i\ cos\,\theta_{ram} = ne\sqrt{\frac{eT}{2\pi m_e}}\,exp\left(\frac{\phi}{T}\right) \qquad (2)$$

For a surface directly facing the ram at a typical low-Earth-orbit speed of 7,500 m/s in a 0.1 eV plasma, the surface is at -0.195 V, twice the temperature.

2) *Flowing Maxwellian*

We can calculate the current to a surface in motion in the orbit-limited approximation (flowing Maxwellian). The orbit-limited approximation is commonly used when modeling tenuous plasmas. It is strictly correct for a 1/r potential field with no physical or electrostatic barriers.[1] It is appropriate for cases in which the Debye length is significantly larger than the spacecraft size, and therefore, while it is often useful as a first estimate, the results must be confirmed by other techniques when used in dense, cold plasma. The current is given by the following expression [2]

$$j = q\int_L^\infty \left(\frac{E}{E\pm\phi}\right)\sqrt{\frac{e}{2\pi}}\,\frac{E}{T}\,n\,exp\left(-\frac{E + mU^2/2 - \sqrt{Em/2}\,U\cos\psi}{T}\right)dE \qquad (3)$$

where the integration variable E represents the energy at the surface, the integral limit L is 0 for the repelled species and $|\phi|$ for the attracted species, the fraction in parentheses represents the orbit limited enhancement or reduction of the current, the upper sign is for ions and the lower sign is for electrons, U is the velocity of the plasma, and ψ is the angle between the flow vector and the incident velocity at infinity, which can be related to the velocity at which the particle strikes the surface. In the absence of motion, this formula reduces to a Maxwellian.

In the orbit-limited approximation, the potential of a 1 cm radius conductive stationary sphere in a 10^{11} m^{-3}, 0.1 eV oxygen plasma (λ_D = 0.7 cm) is -0.36 V, less negative than in the planar approximation. The floating potential of the same sphere moving at a typical low Earth orbit velocity of 7.5 km/sec is -0.324 V, slightly less, as the higher ion current in the ram more than makes up for the lower ion current in the wake.

3) Sheath

In dense plasma with high potentials, the disturbed region is referred to as a sheath. In the sharp sheath edge approximation, the sheath edge represents a demarcation between a low potential exterior region containing neutral undisturbed plasma and a high potential interior region from which one species is excluded. The charge of the attracted species in the sheath region balances the charge on the spacecraft surfaces.

Consider a simple system of two conductive spheres biased with respect to each other. The spheres are 10 cm in radius and their centers are 40 cm apart. Fig. 1 shows the potentials in a plane through their centers with the sphere potentials set to +50 V and -50 V. In this calculation, the plasma is 10^{11} m^{-3}, 0.2 eV giving a Debye length of 1.05 cm, short compared with the sphere size and separation. The ions are atomic oxygen and the spheres are stationary. Within each sheath, the repelled species is excluded leaving behind the charge of the attracted species to balance the charge on the sphere. The overlapping sheaths extend about 10 cm out from the spheres. Outside of that region the potentials are low.

When a sheath forms, the current to an object can be estimated as the current to its sheath. This is the approach used by EWB [3] (Environment WorkBench). In this approximation, each component is surrounded by a sheath of radius r_{sc} where r_{sc} is the solution to the equation

$$\frac{\phi}{T} = 0.8356\left(\frac{r_{sc}}{\lambda_D}\right)^{4/3}\left(\left(\frac{r_{sc}}{a}\right)^{3/4} - \left(\frac{a}{r_{sc}}\right)0.17\right)^{4/3}, \qquad (4)$$

where a is the radius of a sphere with the same surface area as the collecting area of the component. This formula is based on earlier work by Parker.[4] The sheath surfaces are then divided

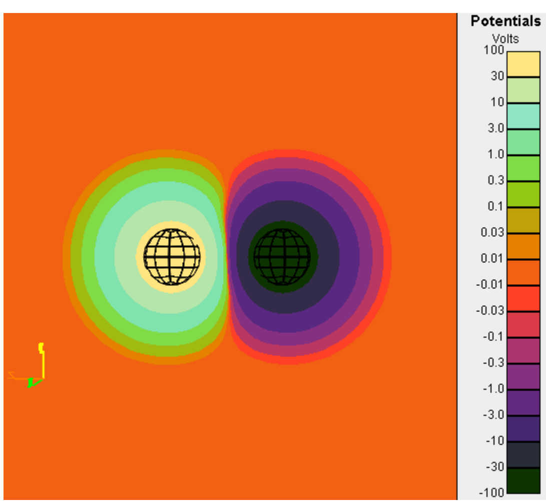

Fig. 1. Illustration of potentials on plane through center of stationary two-sphere object with spheres are +50 V and -50 V, in 10^{11} m^{-2}, 0.2 eV oxygen plasma.

into several parts. Portions of the sheath that overlap are discarded. The current to each sheath segment is determined by the low potential formula above. This approach works well when the problem is sheath dominated. In this model, the floating potentials of the two spheres are -19.6 V and +80.4 V.

B. Tracked currents in potentials computed with analytic space charge

A more computationally intensive approach is to numerically solve Poisson's equation using an analytic formula for the space charge and then track charged particles in the resulting space potentials. One such approximation that is applicable at both high and low potentials is the following non-linear formula[5]

$$\frac{\rho}{\varepsilon_o} = -\frac{\phi}{\lambda_D}\frac{max(1, C(\phi, E))}{1+\sqrt{4\pi}|\phi/T|^{3/2}} \qquad (5)$$

$$C(\phi, E) = min\left((R/r)^2, 3.545|\phi/T|^{3/2}\right)$$

$$(R/r)^2 = 2.29|E\lambda_D/T|^{1.262}|T/\phi|^{0.509}$$

where the symbols refer to the local potential, ϕ, and the local electric field, E. This function smoothly interpolates between linear Debye screening at low potentials and the charge density of a single accelerated and converging species at high potentials. The quantity C, which is a function of the local potential and electric field, accounts for the increase in charge density as charged particles from a large area are attracted to a small region. The convergence formula was developed to fit the results of Langmuir and Blodgett [6] for current collection by a sphere. In a dense plasma, when the spacecraft velocity and Earth's magnetic field have minimal effect on the charge density within the sheath, this formula is appropriate.

Once Poisson's equation has been solved for space potentials, surface currents can be determined by tracking macroparticles either from a sheath edge or from the boundary of the computational space as appropriate.

When a sheath exists, it can be more convenient to track macroparticles from the sheath edge. Macroparticles can be created at the sheath edge and then tracked in the computed potentials and any specified magnetic fields to determine the current to each surface. In the computation, the sheath edge is a surface at a specified potential, for example, $\phi = \pm T\,ln\,2$. The justification for this choice is that because the attracted species is absorbed by the sheath, only the inward moving component is present, comprising half the ambient density. The repelled species, whose density satisfies $n(\phi) = n\,exp(-|\phi/T|)$, must also be at half the ambient density, leading immediately to this sheath potential. The current for each macroparticle is the thermal current for the sheath area it represents. The simplest assumption for the initial velocity of the macroparticles is the average velocity of charged particles crossing the sheath edge, $\sqrt{\frac{2eT}{\pi m}}$.

The approximations made when tracking from a sheath are not valid when either the thermal distribution of particle velocities or the spacecraft velocity is important. In these situations macroparticles are tracked from the boundary of the computational grid. Enough macroparticles need to be tracked to capture the distribution function. The velocity and current carried by each macroparticle can be assigned stochastically or a specified portion of the distribution function can be explicitly specified. Macroparticles can be distributed so that they have approximately the same weight or macroparticles with less current can be used to represent the edges of the distribution function.

A geometric wake factor can be added to the charge density and sheath current formulas to account for spacecraft (and/or plasma) motion.

The stationary two-sphere example discussed above can be used to illustrate this type of model. The potentials shown in Fig. 2 were computed by *Nascap-2k* using the charge density formula of (5). The current to each sphere was computed both by tracking macroparticles from the sheath edge and by tracking particles from the boundary of the computational space and the sphere potentials were adjusted to find the potentials for which the net current is zero. The surface potentials of +0.25 V and -99.75 V, shown in Fig. 2, are those that result when tracking from the sheath. This result is 20 V different than that obtained for the same sphere size and plasma properties using the analytic model of sheath current above.

When tracking from the sheath edge, the current to each sphere includes only the attracted species. When tracking macroparticles of both species from the boundary of the computational space, it is possible to find a floating potential with both spheres negative. For this geometry, the floating potentials of -100.2 V and -0.2 V were found. The -0.2 V sphere collects both ions and the high energy tail of the electron phase space distribution function, which when combined with the ion collection of the -100.2 V sphere gives zero net current.

As shown in Fig. 3, at lower density, the large (negative) sheath gets larger, enveloping the positive sphere. In order to achieve current balance, the positive sphere is at a higher potential, here 13.9 V, so that the positive potential peeks through and electrons can be collected. For this simple geometry, the ratio of the exposed sheath areas matches the ratio of plasma thermal currents.

TABLE II. TWO-SPHERE FLOATING POTENTIALS (V) COMPUTED USING DIFFERENT TECHNIQUES

Debye length (cm)	1.05		3.32		10.5	
Analytic currents, Sheath	-90.9	+9.1	-19.6	+80.4		
Analytic currents, Maxwellian	-99.6	+0.4	-99.6	+0.4	-99.6	+0.4
Tracked currents from sheath	-99.75	+0.25	-98.1	+1.9	-86.1	+13.9
Tracked currents from boundary	-100.2	-0.2	-98.7	+1.3	-88	+12

The floating potentials for the two-sphere example for 10^{11} m^{-3}, 10^{10} m^{-3} and 10^{9} m^{-3}, 0.2 eV, oxygen plasmas are shown in Table II. The results using the Analytic Currents, Sheath model matches the other results best in the shortest Debye length case, for which it is expected to be most accurate. For this example, the results are not sensitive to whether the currents are tracked from the sheath edge or the boundary. When the spacecraft is moving and when the currents themselves are important (rather than just their relative values), it can be important to track the macroparticles from the boundary.

C. *Hybrid PIC*

When the spacecraft geometry is complex enough that analytic charge densities are inadequate, the charge density can be given by the sum of an ion density determined by tracking macroparticles and an electron density given by a Boltzmann function. The equilibrium electron density is Boltzmann for negative potentials in the absence of a potential barrier and nearly Boltzmann for small barriers.

As the ion macroparticles are used to give the ion charge density, they must be tracked from the computational boundary.

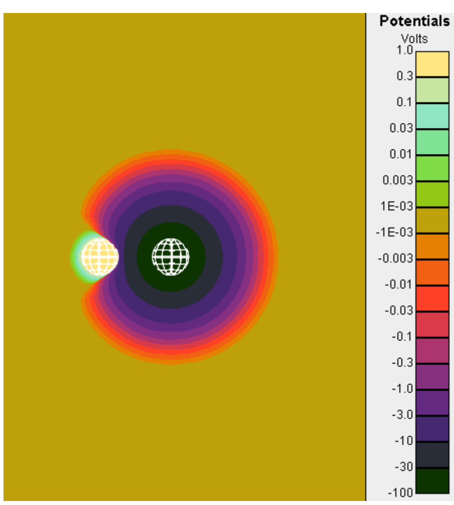

Fig. 2. Potentials on plane through center of stationary two-sphere object with spheres at their floating potentials, of +0.25 V and -99.75 V, computed by tracking macroparticles from the boundary of the computational space, in 10^{11} m^{-3}, 0.2 eV plasma.

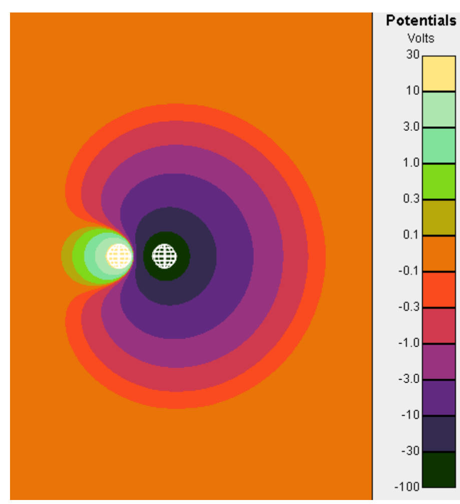

Fig. 3. Potentials on plane through center of two-sphere object with spheres at their floating potentials of +13.9 V and -86.1 V, in 10^{9} m^{-3}, 0.2 eV plasma (λ_D=10.5 cm).

To get self-consistent potentials and ion densities, there are two approaches. The ions can be tracked throughout the grid and then the resultant potentials computed and the process iterated, possibly including sharing of ion densities between iterations. Alternatively, a more traditional PIC approach can be used in which the computational space is filled with ion macroparticles. The macroparticles are tracked for a short time, the potentials are computed, additional ions are injected from the computational boundary, and the process repeated until all transients disappear. In both cases, the time required to reach equilibrium depends on how close the initial potentials are to the potentials for zero current. As mentioned above for electron tracking to obtain surface currents, the initial velocities of the macroparticles used to represent the velocity distribution of phase space can be assigned stochastically or a specified portion of the distribution function can be explicitly specified. The accuracy of the potential solution (and therefore the resulting currents and zero current surface potentials) is limited by the adequacy of sampling of the ion phase space distribution function.

The tracked ion macroparticles can be used to compute the surface currents as well as the spatial charge density. The electron surface current can be computed using any of the analytic or tracked approaches described above.

The importance of velocity resolution can be illustrated by considering the floating potential of an isolated, conducting sphere. While an isolated sphere or cube is a favorite for testing charging codes, its symmetry and low potential make it numerically challenging. We use a stationary, isolated 1 cm radius conductive sphere. In a 10^{10} m^{-3} to 10^{12} m^{-3}, 0.1 eV, oxygen plasma, the Debye length of 0.235 cm to 2.35 cm is comparable to the sphere size. Above we obtained a floating potential of −0.51 V in the planar limit and −0.36 V in the orbit-limited current collection limit. Table III gives the floating potential of this sphere computed using an analytic charge density and Hybrid PIC techniques with various representations of the velocity distribution.

First, we use the analytic charge density function when computing the space potentials, track a distribution of ion macroparticles in those potentials from the boundary of the computational space, and use the Boltzmann relation to specify the electron current. When the Debye length is larger than the sphere, as expected, the orbit limited result is obtained. In a shorter Debye length plasma, the surface potential is shielded by the attracted ions, so that ion collection is less than orbit-limited, and the sphere floats more negative. Fig. 4 shows the potential in a plane through the center of the sphere.

If the same calculation is performed using the Hybrid PIC approach with a minimal distribution of initial velocity values of the ion macroparticles, the resulting sphere potential is slightly more negative and the space potentials seen in Fig. 5 result. In this calculation, the thermal velocity distribution is represented by eight evenly weighted macroparticles with the average thermal velocity plus a component to represent the thermal distribution about a randomly generated coordinate axis. That the lowest potential contour is somewhat square, like the computation grid, rather than circular, like the sphere, suggests that the results may not be correct. Adding additional ion macroparticles with a wider range of initial velocities, shown in Fig 6 gives a result similar to the analytic charge density calculation (which for this calculation is expected to give a highly reliable result). A total of 64 macroparticles were created at each initial location. In each of the three spatial directions, these macroparticles represent the 10% of ions moving fastest in the negative direction, the other 40% moving in the negative direction, 10% of the ions moving fastest in the positive direction, and the other 40% moving in the positive direction.

TABLE III. FLOATING POTENTIAL (V) OF SPHERE COMPUTED USING DIFFERENT TECHNIQUES

Debye length (cm)	0.24	0.74	2.35
Analytic charge density, tracked ion current and analytic electron current	-0.40	-0.37	-0.36
Hybrid PIC with poor ion velocity resolution		-0.40	
Hybrid PIC with better ion velocity resolution	-0.38	-0.37	-0.36
Hybrid PIC and track electrons with poor velocity resolution for currents	-0.33	-0.30	-0.28
Hybrid PIC and track electrons with better velocity resolution	-0.37	-0.34	-0.33

When the electron current is computed using macroparticles, the adequacy of the representation of the high energy portion of phase space becomes very important, as can be seen by comparing the floating potentials computed with poor and better resolution. In the first of the calculations in which the electron current is determined by tracking electron macroparticles, the electron thermal distribution is represented in the same manner as the ion distribution. This is inadequate as only a small fraction of the electrons have enough energy to reach the sphere. In the better resolution calculation, a similar scheme for representing the distribution is used. At each emitting point on the boundary, 1,000 macroparticles are created. The electrons distribution is divided into those with the 2.5% most negative velocities, those with the next 2.5% most negative, those with the next 5% most negative, those with the next 5% most negative, the remaining 35% with negative velocities, and the same for positive velocities.

D. PIC

The first principles approach is to use macroparticle tracking of all species to determine the charge density for Poisson’s equation as well as to determine the surface currents. This requires even more computer time and a great deal of skill to avoid numeric pathologies. There is an extensive literature of techniques to help speed solutions and improve stability developed by researchers modelling dynamic plasma conditions. The calculation is inherently dynamic, so averaging and smoothing are generally needed to determine the steady-state solution.

This approach becomes necessary when investigating novel effects and exploring dynamic conditions.

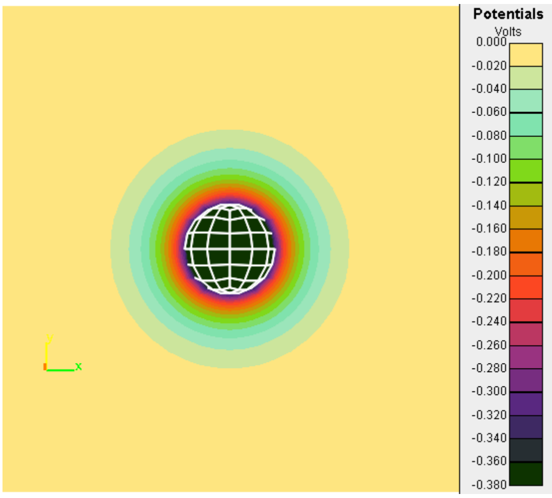

Fig. 4. Potentials in a plane through the center of a sphere computed using the analytic space charge density of (5) at the floating potential of -0.37 V in a 10^{11} m^{-3}, 0.1 eV plasma.

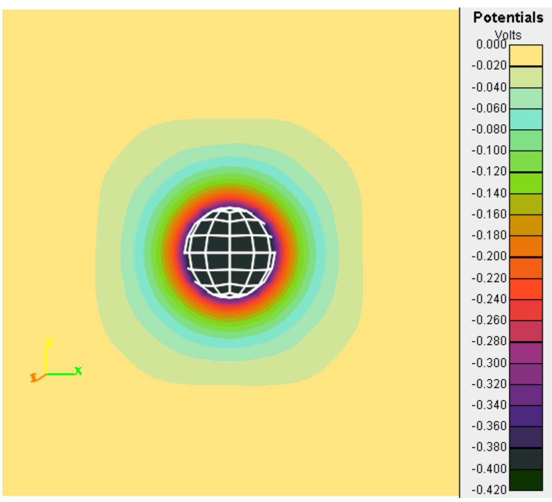

Fig. 5. Potentials in a plane through the center of a sphere computed using the Hybrid PIC method with minimal velocity resolution at the floating potential of -0.40 V in a 10^{11} m^{-3}, 0.1 eV plasma.

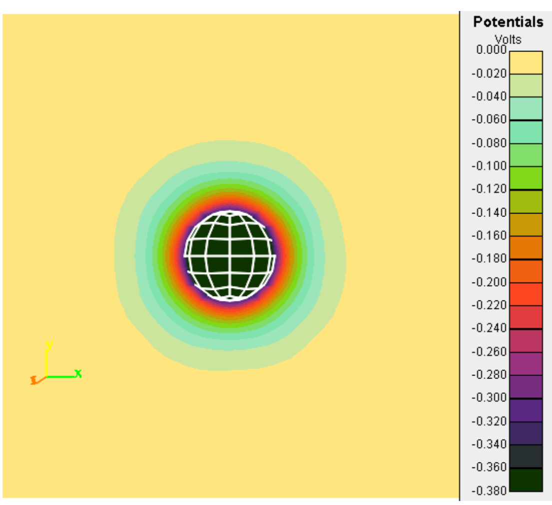

Fig. 6. Potentials in a plane through the center of a sphere computed using the Hybrid PIC method with better velocity resolution at the floating potential of -0.40 V in a 10^{11} m^{-3}, 0.1 eV plasma.

III. Conclusions and Implications

We have surveyed a selection of methods to compute currents when solving for floating potentials of surfaces in dense, cold plasma such as observed in low Earth orbit. The methods range from analytic to full PIC. Analytical methods are easy to apply and give results quickly. However, they are only applicable when the underlying approximations are valid. At the other extreme, full PIC calculations are applicable to complex geometry and dynamic conditions. However, they require large investments of computational resources and a great deal of skill to obtain reliable results.

In practice, computations generally combine analytic methods with PIC techniques. With an understanding of the underlying approximations, reliable results can be obtained with the minimum necessary resources.

The extent to which the approximations are valid is highly dependent on the system and the question at hand. To gain confidence in the results, it is best to use the simplest applicable models to gain insight, and then use more complex approaches to verify the applicability of the approximation. When calculations are dependent on adequate resolution, they can be checked by verifying that the results are the same with improved resolution.

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Spacecraft Charging Technology Conference

Pasadena, California

June 26, 2014

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- Background
- Calculation techniques
  - Analytic
  - Tracked currents/Analytic space charge
  - Hybrid PIC
  - PIC
- Conclusions

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Charging in Dense, Cold Plasma

- High currents => Charging in milliseconds
- Generally interested in steady-state
- Net current from incident charged particles only
  - Secondary yield negligible
  - Photoemitted electron current generally much smaller than incident current
- Typical low-Earth-orbit velocity of 7 km/sec
  - Mesothermal plasma: $v_{th}^{e} > v^{sc} > v_{th}^{i}$
  - Wake charging can be slow
- Potentials
  - Up to a few times the plasma temperature
  - Or of order the ion kinetic energy
  - Or spacecraft generated, such as due to exposed solar cell interconnects and bus bars

Temperature	0.1 to 5 eV
Density	10^9 to 10^{12} m^{-3}
Debye length	0.2 to 50 cm
Electron thermal current	9 μA m^{-2} to 60 mA m^{-2}
Oxygen thermal current	0.05 μA m^{-2} to 0.3 mA m^{-2}
Hydrogen thermal current	0.2 μA m^{-2} to 1 mA m^{-2}
Ram Oxygen current	1 μA m^{-2} to 1 mA m^{-2}

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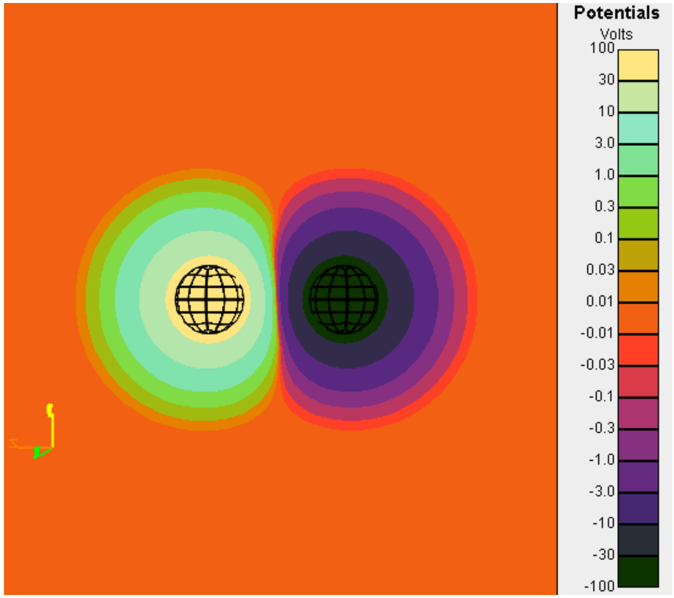
Spacecraft Generated Voltages And Sheaths

Simplified Example

- Geometry
  - Two conductive spheres
  - 10 cm in radius and 40 cm apart
  - Biased 100 V with respect to each other
- Plasma
  - 10^{11} m^{-3}, 0.2 eV
  - 1.05 cm Debye length
- Debye length << Sphere size
- Dense plasma shield potentials
- Intersecting sheaths form
  - Low potential exterior undisturbed plasma
  - High potential interior with one species excluded
  - At ±50 V
    - Ion current of 0.01 mA and electron current of -1.8 mA
    - Net negative current

Potentials set at ±50 V



Calculating Charging in Cold, Dense Plasma

- General procedures
  - Current balance: Compute net current as a function of surface potentials and find zero of IV curve
  - Charging: Assume surface potentials and iteratively compute currents and change in surface potentials using assumed capacitances
- Current collection approximations
  - Analytic currents
    - Currents computed using a formula
  - Particle tracking/Analytic space charge
    - Poisson's equation solved with space charge specified by an analytic formula
    - Current of one or both species computed by tracking macroparticles
  - Hybrid PIC
    - Poisson's equation solved with space charge given by ion densities from macroparticle tracking and electron densities from an analytic formula
    - Current of one or both species computed by tracking macroparticles
  - PIC
- Review benefits and drawbacks

Analytic (1/2)

- Assume an idealized geometry and calculate an exact answer
  - Benefits
    - Once the formula is developed, applying it to new systems is quick and easy
  - Drawbacks
    - Limited by how far system under consideration is from ideal
- Planar
  - $$eN\sqrt{\frac{eT}{2\pi m_i}} = eN\sqrt{\frac{eT}{2\pi m_e}}\exp\left(\frac{\phi}{T}\right)$$
    - Attracted current is ion random thermal current
    - Repelled current is electron random thermal current reduced by Boltzmann
    - Isolated surface in 0.1 eV Oxygen plasma floats at -0.51 V
  - In ram flow

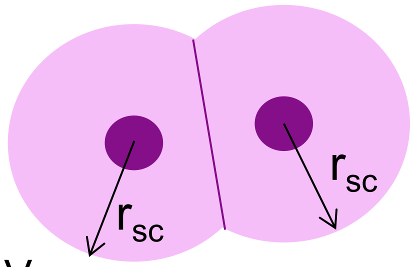
$$eNv_i\cos\theta_{ram} = eN\sqrt{\frac{eT}{2\pi m_e}}\exp\left(\frac{\phi}{T}\right)$$

Analytic (2/2)

- Flowing Maxwellian
  - Orbit-limited approximation for equilibrium plasma
    - Strictly correct for 1/r to $1/r^2$ potential with no physical or electrostatic barriers
  - 1 cm radius conductive sphere in 10^{11} m^{-3}, 0.1 eV stationary Oxygen plasma (λ_D = 0.7 cm) floats at -0.36 V
- Overlapping Sheaths (Jongeward, unpublished; used in EWB (Environment WorkBench))
  - Each component
    - Approximate as sphere of radius a with same surface area
    - Sheath of radius r_{sc}

$$\frac{\phi}{\theta} = 0.8356\left(\frac{r_{sc}}{\lambda_D}\right)^{4/3}\left(\left(\frac{r_{sc}}{a}\right)^{3/4} - \left(\frac{a}{r_{sc}}\right)0.17\right)^{4/3}$$

  - Discard portions of sheaths that overlap
  - Floating potentials for two sphere example: -90.9 V and 9.1 V
  - Most reliable for high voltages in short Debye length plasma



Tracked Currents/Analytic Space Charge (1/3)

- Numerically solve Poisson's equation using an analytic formula for the space charge and track macroparticles in resulting potentials
  - Benefits
    - Accounts for physical and electrostatic barriers to current collection
    - Low computational overhead
  - Drawbacks
    - Requires some expertise in constructing computational space
    - Limited by approximations made in analytic formula for space charge
    - Numerical techniques required for potentials solution with Debye length less than volume element size
- Example space-charge formula
  - $$\frac{\rho}{\varepsilon_o} = -\frac{\phi}{\lambda_D}\,\frac{\max\left(1, C(\phi, E)\right)}{1+\sqrt{4\pi}\left|\phi/\theta\right|^{3/2}}$$

    $$C(\phi, E) = \min\left((R/r)^2, 3.545\left|\phi/\theta\right|^{3/2}\right)$$

    $$(R/r)^2 = 2.29\left|E\lambda_D/\theta\right|^{1.262}\left|\theta/\phi\right|^{0.509}$$
  - Linear Debye screening at low potentials and charge density of single accelerated and converging species at high potentials (*Nascap-2k* Scientific Documentation)
  - "C" accounts for convergence and is a fit to the results of Langmuir and Blodgett for a sphere

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Tracked Currents/Analytic Space Charge (2/3)

- Track from sheath edge
  - As attracted species is absorbed, only inward moving component is present. Quasi-neutrality gives a sheath potential of $\phi_s = \pm\theta \ln(n/n_o) = \pm\theta \ln(0.5)$
- Track from computational boundary
  - Required
    - in absence of a sheath
    - when thermal distribution of particle velocities important
    - when spacecraft velocity is important
  - Tracked macroparticles must adequately represent the thermal distribution function
- Geometric wake factor can be added to the charge density and sheath current formulas to account for spacecraft and/or plasma motion

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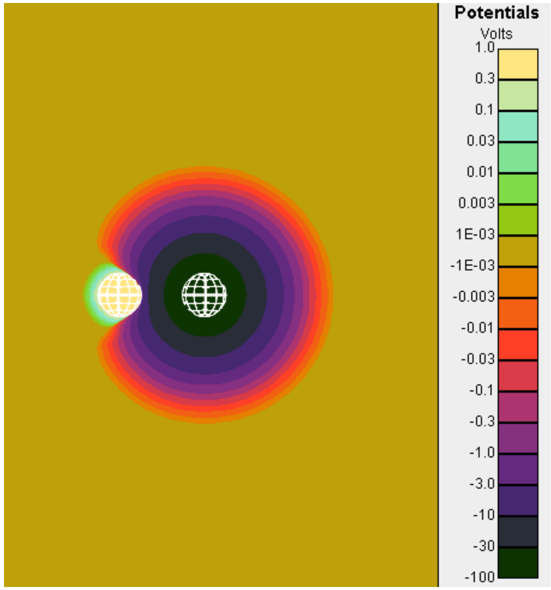
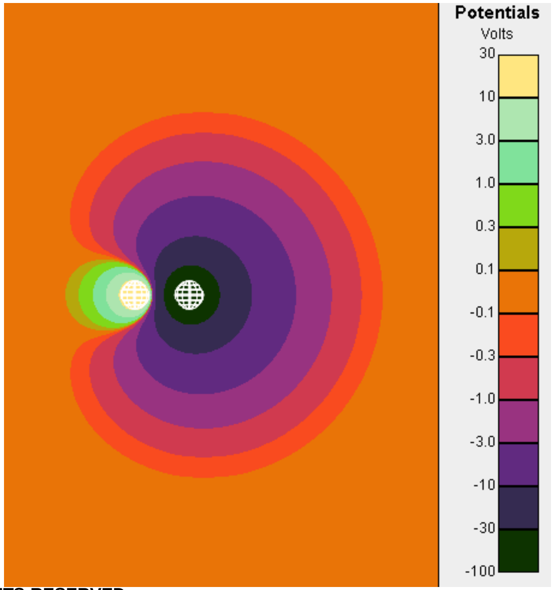
Tracked Currents/Analytic Space Charge (3/3)

- Track from sheath: Two sphere example
  - In lower density plasma, large (negative) sheath is larger, so positive sphere needs to be at higher potential to “peek through”
  - Large sheath envelops small sheath so that ratio of exposed sheath areas matches the ratio of plasma thermal currents

10^{11} m^{-3}, 0.2 eV plasma
0.2 V and -99.7 V



10^{9} m^{-3}, 0.2 eV plasma
13.9 V and -86.1 V



Hybrid PIC (1/2)

- Numerically solve Poisson's equation with the ion space-charge from particle tracking and the electron space-charge from a formula
  - Benefits
    - Necessary when analytic charge density approximation invalid
    - Less computationally intensive than full PIC
  - Drawbacks
    - Like with analytic space-charge, requires expertise in constructing computational space
    - Requires more sophisticated numeric techniques for stability
    - Tracked macroparticles must adequately represent the thermal distribution function
- For negative potentials in the absence of a potential barrier, equilibrium electron density is strictly a thermal Boltzmann distribution in a closed system

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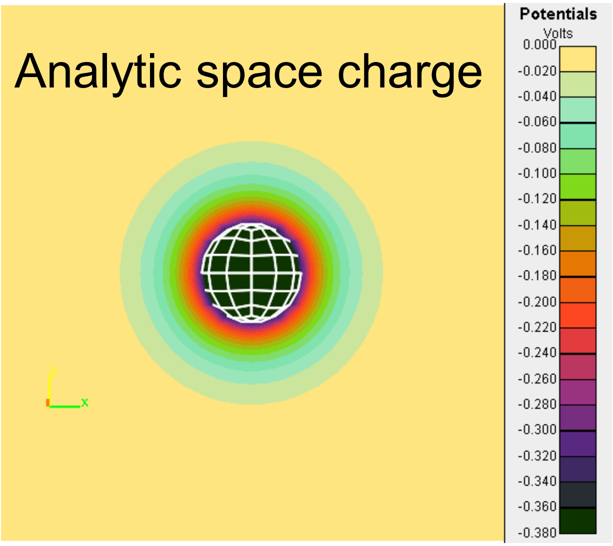
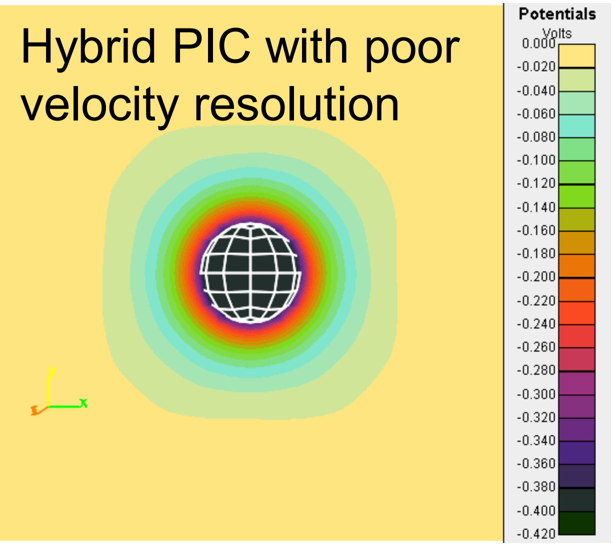
Hybrid PIC (2/2)

- Hybrid PIC example: 1 cm radius sphere
- Analytic results
  - Planar: -0.51 V
  - Orbit limited (Flowing Maxwellian): -0.36
- Adequate thermal resolution needed
- Floating potential of small sphere (few times plasma Debye length) near orbit limited result

Debye length (cm)	0.24	0.74	2.35
Analytic space charge	-0.40	-0.37	-0.36
Hybrid PIC (inadequate velocity resolution)		-0.40	
Hybrid PIC (good ion velocity resolution)	-0.38	-0.37	-0.36
Hybrid PIC (track electrons for currents—inadequate electron velocity resolution)	-0.33	-0.30	-0.28
Hybrid PIC (track electrons for currents—better electron velocity resolution)	-0.37	-0.34	-0.33





PIC

- First principles approach uses macroparticle tracking of all species to determine the charge density for Poisson's equation as well as to determine the surface currents.
  - Benefits
    - No approximations (except as needed for stability or speed of solution)
    - Useful when investigating novel effects and exploring dynamic conditions
    - Easy to explain
  - Drawbacks
    - Computationally intensive
    - Requires highly skilled users and programmers to avoid numeric pathologies
    - Mitigation of drawbacks: Extensive literature of techniques to speed solutions and improve stability developed by researchers modeling dynamic plasmas

Conclusion

- Surveyed approaches to solving for floating potentials of surfaces in dense, cold plasma such as low-Earth-orbit
  - Within the approaches discussed there are a multitude of a variations applicable to different systems
  - Analytic approximations
    - Easy to apply and quick
    - Applicable only when approximations valid
  - PIC
    - Applicable to complex geometry and dynamic conditions
    - Numerically challenging
  - Many techniques combine analytic approximations with PIC techniques
- Confidence requires using more than one technique and understanding the limitations of each
- Best technique depends on system and question at hand